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Polarization Response Functions and the (e,e′p) Reaction

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In this paper we focus on the $(\vec{e}, e'\vec{N})$ reaction where polarized electrons are used to eject polarized nucleons from an unpolarized nucleus.[46] This reaction has several advantages as a means for increasing the available information necessary to constrain theory. The additional measurable quantities are discrete spin degrees of freedom which can be accessed by providing a polarized electron beam and/or using a polarimeter for the ejected nucleons. Both of these elements exist and the advent of the coming generation of high duty factor electron accelerators should make possible their simultaneous use in coincidence experiments. The discreteness of the spin degrees of freedom can also be used to minimize systematic experimental errors by allowing all of the continuous kinematical variables to be fixed while the spin of the beam is flipped. While this is also true of coincidence experiments using polarized targets, the measurement of ejectile spin circumvents the difficulties of producing polarized targets which can be used in a high current electron beam. From the theoretical standpoint, the $(\vec{e}, e'\vec{N})$ reaction provides direct access to the spin response of the nuclear system. This is, of course, of considerable importance since the strong interactions of the nuclear system are explicitly spin dependent as is the electromagnetic interaction of the electrons with the hadrons of the nucleus. There is, by inference from recent developments in elastic proton scattering,[47,48,49] from the electrodisintegration of the deuteron,[50] and from the unexpected results of longitudinal/transverse separations in inclusive quasielastic electron scattering[1–8] every reason to believe that the addition of these spin observables will considerably constrain the various elements of models of quasielastic electron scattering.

In a previous paper,[46] we presented a formal framework for the description of the $(\vec{e}, e'\vec{N})$ reaction which provides a direct generalization of the usual description of the unpolarized reaction and which treats the spin of the ejected nucleon in a manner consistent with that used in elastic proton scattering. This framework was constructed to explicitly display the dependence of the differential cross section on the polarization of the ejected nucleon. In that paper, we presented a discussion of the constraints placed on the eighteen response functions (thirteen of which depend on ejectile spin) by various symmetries. We also provided limited preliminary

2. Review of Formalism

By virtue of general symmetry principles, the differential cross section for the $(\vec{e}, e'\vec{N})$ reaction, when the residual nucleon is left in its ground state or some discrete excited state, can be written as[46]

$$
\begin{aligned}
&\left(\frac{d^5\sigma}{d\epsilon_{k'} d\Omega_{k'} d\Omega_{p'}}\right)_{h,s'} = \frac{m\,|\vec{p}\,'|}{2\,(2\pi)^3}\left(\frac{d\sigma}{d\Omega_{k'}}\right)_{Mott} \\
&\times\Big\{V_L\,(R_L + R_L^n S_n) + V_T\,(R_T + R_T^n S_n) \\
&+V_{TT}\left[(R_{TT} + R_{TT}^n S_n)\cos 2\beta + \left(R_{TT}^l S_l + R_{TT}^t S_t\right)\sin 2\beta\right] \\
&+V_{LT}\left[(R_{LT} + R_{LT}^n S_n)\cos\beta + \left(R_{LT}^l S_l + R_{LT}^t S_t\right)\sin\beta\right] \\
&+hV_{LT'}\left[(R_{LT'} + R_{LT'}^n S_n)\sin\beta + \left(R_{LT'}^l S_l + R_{LT'}^t S_t\right)\cos\beta\right] \\
&+hV_{TT'}\left(R_{TT'}^l S_l + R_{TT'}^t S_t\right)\Big\}
\end{aligned}
\tag{2.1}
$$

where $\vec{k}$ and ϵ_k ($\vec{k}'$ and $\epsilon_{k'}$) are the momentum and energy of the incident (scattered) electron, h is the incident electron helicity, θ is the electron scattering angle and m is the nucleon mass. Defining $\vec{q}$ and ω as the momentum and energy transfer from the electron, and $Q^2 = -q^2 = \vec{q}^{\,2} - \omega^2$, the Mott cross section is

$$
\left(\frac{d\sigma}{d\Omega_{k'}}\right)_{Mott} = \left(\frac{\alpha\cos\theta/2}{2\epsilon_k\sin^2\theta/2}\right)^2 = \left(\frac{2\alpha\epsilon_{k'}\cos\theta/2}{Q^2}\right)^2 . \tag{2.2}
$$

The kinematic factors are defined as

$$
\begin{aligned}
V_L &= \frac{Q^4}{\vec{q}^{\,4}} \\
V_T &= \left(\frac{Q^2}{2\vec{q}^{\,2}} + \tan^2\frac{\theta}{2}\right) \\
V_{TT} &= \frac{Q^2}{2\vec{q}^{\,2}} \\
V_{LT} &= \frac{Q^2}{\vec{q}^{\,2}}\left(\frac{Q^2}{\vec{q}^{\,2}} + \tan^2\frac{\theta}{2}\right)^{\frac{1}{2}}
\end{aligned}
\tag{2.3}
$$

on the final state boundary condition, incoming $(-)$ and outgoing $(+)$ scattered waves. For the $(\vec{e}, e'\vec{p})$ reaction, the $(-)$ condition is appropriate. In the general case, (2.5) does not imply an immediately useful result. However, in cases where the boundary conditions can be ignored, (2.5) states that the symmetric part of $W^{\mu\nu}\left(\hat{s}'_R\right)$ is independent of $\hat{s}'_R$ and that the antisymmetric part of $W^{\mu\nu}\left(\hat{s}'_R\right)$ is proportional to $\hat{s}'_R$. This is because the dependence of $W^{\mu\nu}\left(\hat{s}'_R\right)$ on $\hat{s}'_R$ is at most linear. These results really apply only in idealized models where there is no scattered wave at infinity, such as the PWIA. However, the PWIA is a useful conceptual limit so that the behavior of the response functions under such conditions is of interest. In the column of Table 1 headed "TP", response functions which survive the conditions (2.5) when the boundary conditions are ignored are labeled "even" while those which do not are labelled "odd".

Additional properties of the response functions are of interest from the experimental viewpoint. Since it is more difficult to measure ejected nucleons which have momenta lying out of the electron scattering plane (because it is necessary to move either the electron beam or the hadron spectrometer out of the plane of the floor of the experimental hall), it is useful to know which of the response functions can be detected in the electron scattering plane ($\beta = 0$ or $\beta = \pi$) and which require going out of plane. The column of Table 1 labeled "Survives In-Plane" indicates which response functions contribute in the electron scattering plane.

Coincidence experiments are often performed with the ejectile momentum parallel ($\alpha = 0$) or antiparallel ($\alpha = \pi$) to the momentum transfer. This is the so-called parallel-antiparallel kinematics, where the parallel/antiparallel distinction is conventionally made according to whether[9] the recoil momentum $\vec{P}_R$ is parallel or antiparallel to $\vec{q}$ or, alternately, whether[12] the "missing" momentum $\vec{p}_m = -\vec{P}_R$ is parallel or antiparallel to $\vec{q}$ (in PWIA, only, $\vec{p}_m$ is the initial nucleon momentum). The column of Table 1 labeled "Survives in Parallel Kinematics" indicates which of the response functions contribute to the cross section under these kinematical conditions. The general definitions of the response functions depend upon the system in this limit. The unit vector $\hat{l}$ is chosen to point along $\vec{p}'$, $\hat{n}$ points in the positive y-direction and $\hat{t} = \hat{n} \times \hat{l}$. This corresponds to the limiting process of first taking the limit $\beta \rightarrow 0$ and then $\alpha \rightarrow 0(\pi)$, that is, the natural spin coordinate system in parallel kinematics is defined by simply allowing $\beta = 0$ in Fig. 1. Thus the spin-dependent response functions R^n_{LT}, $R^t_{LT'}$ and $R^l_{TT'}$ determine the transverse component normal to the electron scattering plane, transverse component in the electron scattering plane, and the longitudinal component of the ejectile polarization vector. Note from (2.1) that the detection of the in-plane components of the polarization vector require a polarized electron beam whereas the normal component does not. Because the response functions corresponding to the in-plane polarizations are predicted to be large (see later discussion) and because those corresponding to the normal polarization are predicted to be small, but dynamically sensitive, ejectile polarization measurements in the case of parallel/antiparallel kinematics appear to be a promising means of extracting dynamical information about the $(\vec{e}, e'\vec{p})$ reaction process.

Since the polarization of existing electron beams is limited to about 40 percent and such beams have limited current, coincidence experiments which do not require beam polarization can be performed more rapidly. The column labeled "Electron Polarization Required" in Table 1indicates which of the response functions contribute only when the beam is polarized.

Finally, terms contributing to the cross section in the electron scattering plane with a contribution to the cross section which changes sign under the change in kinematics $\beta = 0 \rightarrow \beta = \pi$ can be most easily separated from the total cross section, at least in principle, by making the change in azimuthal angle and subtracting cross sections. The last column of Table 1, labeled "Reflection Symmetry", gives the symmetry of the contribution to the cross section associated with each of the response functions under the reflection of the ejectile momentum $\vec{p}'$ through the yz-plane (irrespective of any associated direction changes of $\hat{n}$, $\hat{l}$, and $\hat{t}$). The terms of special interest are those with odd symmetry which contribute in the electron scattering plane.

and

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2) \tag{3.4}$$

with $\tau = Q^2/(4m^2)$. Note that (3.3) can be derived by a nonrelativistic expansion of the free Dirac current matrix elements to order $(1/m)$. As long as the lowest order is sufficient, there are no ambiguities in (3.2) associated with the use of the Gordon identity. There is, however, some ambiguity associated with the choice of form factors in (3.2). From (3.3) it is clear that the form factor $F_1(Q^2)$ differs from the Sachs form factor $G_E(Q^2)$ by a term proportional to τ. Since τ is manifestly of order $(1/m)^2$, the difference between the two form factors is of higher order than is retained in the nonrelativistic expansion. Under circumstances where the difference between $F_1(Q^2)$ and $G_E(Q^2)$ becomes quantitatively significant, the use of this lowest-order expansion clearly becomes invalid, and higher-order corrections must be carefully treated.

The expression for the nuclear response tensor can be simplified by noting that

$$\begin{aligned}\sum_m \Psi_{nljm}\left(\vec{p}\,' - \vec{q}\right)\Psi^\dagger_{nljm}\left(\vec{p}\,' - \vec{q}\right) =& \frac{1}{2} n_{nlj}\left(\left|\vec{p}\,' - \vec{q}\right|\right) \\ =& \frac{(2j+1)}{8\pi} R^2_{nlj}\left(\left|\vec{p}\,' - \vec{q}\right|\right)\end{aligned} \tag{3.5}$$

where $n_{nlj}\left(\left|\vec{p}\right|\right)$ is the momentum density distribution for a proton (neutron) in the nlj subshell and R_{nlj} is the single-particle radial wave function. The nuclear response tensor therefore reduces to

$$W^{\mu\nu}\left(\hat{s}'_R\right) = Tr\left[\frac{1}{4}\left(1 + \vec{\sigma}\cdot\hat{s}'_R\right) J^\nu\left(p', q\right) J^{\mu\dagger}\left(p', q\right)\right] n_{nlj}\left(\left|\vec{p}\,' - \vec{q}\right|\right). \tag{3.6}$$

Performing the traces for various combinations of the currents and comparing to (A4), the response functions are

$$\begin{aligned}R_L =& F_1^2(Q^2) n_{nlj}\left(\left|\vec{p}\,' - \vec{q}\right|\right) \\ R_L^n =& 0\end{aligned}$$

It is straightforward to generalize to a Dirac plane wave approximation. In this approximation, the nuclear response tensor can be written as

$$W^{\mu\nu}\left(\hat{s}'_R\right) = \sum_{m,s''} \overline{u}\left(\vec{p}\,', s''\right) \frac{1}{2}\left(1+\gamma_5 \not{s}'\right) \Gamma^{\nu}(q) \Psi_{nljm}\left(\vec{p}\,' - \vec{q}\right) \times \overline{\Psi}_{nljm}\left(\vec{p}\,' - \vec{q}\right) \overline{\Gamma}^{\mu}(q) u\left(\vec{p}\,', s''\right) \tag{3.8}$$

where

$$s' = \left(\frac{\vec{p}\,' \cdot \hat{s}'_R}{m}, \hat{s}'_R + \frac{\left(\vec{p}\,' \cdot \hat{s}'_R\right)\vec{p}\,'}{m\left(E' + m\right)}\right), \tag{3.9}$$

$$\Gamma^{\mu}(q) = F_1(Q^2)\gamma^{\mu} + \frac{F_2(Q^2)}{2m} i\sigma^{\mu\alpha} q_{\alpha} \tag{3.10}$$

and $\overline{\Gamma}^{\mu} = \gamma^0 \Gamma^{\mu\dagger} \gamma^0$ denotes the Dirac adjoint. Again, the choice of the current operator as the usual form of the free Dirac current is somewhat arbitrary. The general form of the fully-off-shell current operator can be constructed using general symmetry arguments and the properties of the Dirac γ-matrices. From four-momentum conservation there are only two independent four-momenta at the vertex. Using any two four-momenta along with the γ-matrices and their commutation relations, it can be shown that there are twelve independent four-vector forms which can be constructed in the Dirac space and which transform properly under parity.[51] In constructing these twelve forms, all scalars involving four-momenta contracted with γ-matrices are incorporated, leaving three remaining momentum-space scalars which can be constructed using only the two independent four-momenta. Each of the twelve four-vectors in the Dirac space is therefore multiplied in general by a form factor which is an arbitrary function of these three scalars. For example, the form factors can be chosen to be functions of the invariant masses of the photon and the two nucleons which join at the vertex. The commutation relations for the γ-matrices can be used to construct generalized Gordon identities which allow for the rearrangement of the various contributions to the vertex functions. In order to uniquely determine the complete off-shell behavior of the vertex function, it is necessary to have a dynamical theory for the nucleon. It may be possible, however, to place some constraints on the vertex be written as

$$W^{\mu\nu}\left(\hat{s}'_R\right) =$$
$$Tr\left[\frac{1}{2}\left(1+\gamma_5 \not{s}'\right)\Gamma^\nu(q)\frac{1}{4}\left(\not{n}^V_{nlj}\left(\vec{p}\,'-\vec{q}\right)+n^S_{nlj}\left(\left|\vec{p}\,'-\vec{q}\right|\right)\right)\overline{\Gamma}^\mu(q)\frac{\not{p}'+m}{2m}\right]. \tag{3.13}$$

The calculation of the various response functions from (3.13) is straightforward, although very tedious, and the resulting expressions are exceedingly complicated. There is, however, a particular case in which the results are both simpler and interesting from a pedagogical standpoint. If the bound state Dirac equation is projected onto the positive energy (plane wave basis) space, eliminating all coupling to the negative energy space, the solution of the Dirac equation can be written as the spinor wave function

$$\Psi_{nljm}\left(\vec{p}\right) = \left(\frac{E\left(\vec{p}\right)+m}{2E\left(\vec{p}\right)}\right)^{1/2}\begin{pmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{E(\vec{p})+m}\end{pmatrix}\Phi_{nljm}\left(\vec{p}\right) \tag{3.14}$$

where the wave function is normalized such that

$$\int d^3p\,\overline{\Psi}_{nljm}\left(\vec{p}\right)\gamma^0\Psi_{nljm}\left(\vec{p}\right) = \int d^3p\,\Phi^\dagger_{nljm}\left(\vec{p}\right)\Phi_{nljm}\left(\vec{p}\right) = 1. \tag{3.15}$$

This simplification eliminates the dynamical aspects of relativity inherent in the Dirac equation. That is, the effects of coupling to virtual negative energy states have been eliminated while the relativistic kinematics of the Dirac equation have been retained. It is in this sense that we refer to equivalent nonrelativistic calculations, as we did in Ref. 33.

In this case the momentum distributions are given by

$$\begin{aligned} n^{V0}_{nlj}\left(\vec{p}\right) &= n_{nlj}\left(\left|\vec{p}\right|\right) = \frac{2j+1}{4\pi}\left|R_{nlj}\left(\left|\vec{p}\right|\right)\right|^2 \\ \vec{n}^V_{nlj}\left(\vec{p}\right) &= \frac{\vec{p}}{E\left(\vec{p}\right)}n_{nlj}\left(\left|\vec{p}\right|\right) \\ n^S_{nlj}\left(\left|\vec{p}\right|\right) &= \frac{m}{E\left(\vec{p}\right)}n_{nlj}\left(\left|\vec{p}\right|\right) \end{aligned} \tag{3.16}$$

where $R_{nlj}\left(\left|\vec{p}\right|\right)$ is the radial part of $\Phi_{nljm}\left(\vec{p}\right)$. With these definitions of the

$$
\begin{aligned}
R_{LT'} &= 0 \\
R^n_{LT'} &= \left\{ F_1(Q^2) + F_2(Q^2)\frac{E'\overline{\omega} - |\vec{q}\,|\,|\vec{p}\,'|\cos\alpha}{2m^2} \right\} G_M(Q^2)\frac{|\vec{q}\,|}{m} n^S_{nlj}\left(|\vec{p}\,' - \vec{q}\,|\right) \\
R^l_{LT'} &= -\left\{ F_1(Q^2)\frac{E'}{m} + F_2(Q^2)\frac{\overline{\omega}}{2m} \right\} G_M(Q^2)\frac{|\vec{q}\,|}{m} \sin\alpha\, n^S_{nlj}\left(|\vec{p}\,' - \vec{q}\,|\right) \\
R^t_{LT'} &= -\left\{ F_1(Q^2)\cos\alpha + F_2(Q^2)\frac{E'\overline{\omega}\cos\alpha - |\vec{q}\,|\,|\vec{p}\,'|}{2m^2} \right\} G_M(Q^2)\frac{|\vec{q}\,|}{m} n^S_{nlj}\left(|\vec{p}\,' - \vec{q}\,|\right) \\
R^l_{TT'} &= \left\{ F_1(Q^2)\frac{|\vec{p}\,'|\,|\vec{q}\,| - \overline{\omega}E'\cos\alpha}{m^2} - F_2(Q^2)\frac{\overline{q}^2}{2m^2}\cos\alpha \right\} G_M(Q^2) n^S_{nlj}\left(|\vec{p}\,' - \vec{q}\,|\right) \\
R^t_{TT'} &= \left\{ F_1(Q^2)\frac{\overline{\omega}}{m} + F_2(Q^2)\frac{E'\overline{q}^2}{2m^3} \right\} G_M(Q^2)\sin\alpha\, n^S_{nlj}\left(|\vec{p}\,' - \vec{q}\,|\right) .
\end{aligned}
$$

where $\overline{\tau} = \overline{Q}^2/(4m^2) = -\overline{q}^2/(4m^2)$. Since these results do not contain the physics of coupling to the negative energy Dirac space in the bound state, we refer to this as the semi-relativistic plane wave impulse approximation (SRPWIA). First, we note that, given the ambiguity in using $G_E(Q^2)$ or $F_1(Q^2)$, equations (3.18) reduce to the comparable expressions in (3.7) in the limit where the momenta are small compared to the nucleon mass. Equations (3.18) exhibit a diversity of dependence on the ingredients from which the response functions are formed that goes a bit beyond that found in the nonrelativistic limit (3.7). The additional structure in (3.18) due to relativistic effects arises exclusively from higher order terms in the $(1/m)$ expansion of the current operator. Further, purely off-shell, terms appear for other choices of the relativistic current operator, yielding different and much more complex forms for the response functions. Thus, the relativistic corrections exhibited in (3.18) must be regarded as representative, only. Of course, model dependence introduced by ambiguities in the off-shell contributions bears directly on the degree of precision with which predictions can be reliably made.

momentum distribution. The effect of such exchange currents can be most clearly identified where the impulse approximation gives a small result, such as at large recoil momentum where the impulse approximation is suppressed by the one-body momentum distribution, whereas momentum sharing allows the effective current operator to make a larger relative contribution. This feature will also be characteristic of other many-body corrections to the effective current operator such as ground state correlations and inelastic rescattering effects.

In addition, the dependence of the effective current operator on the external momenta $\vec{q}$ and $\vec{p}\,'$ is more complicated than that of the free current operator. The functional dependence on the asymptotic momenta and spin of the impulse and many-body contributions can, therefore, be expected to be distinctive. This raises the possibility that kinematical regions may be identified for the various response functions which will tend to emphasize one or more dynamical contributions to the reaction. The additional freedom provided by the measurement of the recoil polarization can be expected to facilitate such attempts to isolate individual physical processes. This has been shown to be the case in existing calculations of electrodisintigration of the deuteron. Thus, although the DWIA results which follow exhibit a number of physically interesting characteristics, this study by no means exhausts the physically interesting issues associated with the $(\vec{e}, e'\vec{p})$ reaction. Considerably more analytical sophistication will be required to fully circumscribe the dynamics relevant to this reaction. DWIA results represent a first step in this direction.

Figures 2-4 display the first comprehensive results of DWIA predictions for the full set of eighteen $(\vec{e}, e'\vec{p})$ response functions. However, before discussing our results it is useful to make some comments on the likely extraction of the information contained in the $(\vec{e}, e'\vec{p})$ cross section. It is clearly a formidable task to undertake the separation of all eighteen response functions. For simple systems such as the deuteron it may be possible for a compressive program of measurements, including target and ejected nucleon polarization, to completely determine the transition current densities up to an overall phase. In this case, ejection of a 135 MeV proton from the $1p_{1/2}$ shell of ^{16}O at a constant momentum transfer of 2.641 fm^{-1}.

Four different dynamical calculations are presented for each of the eighteen response functions in Figs. 2-4. The solid lines represent the Dirac DWIA calculations as described in Ref. 33. These use Dirac optical model scattering wave functions for the ejected nucleon using the Dirac optical potential of Ref. 49, Dirac-Hartree independent particle bound state wave functions[38] and the free Dirac current operator as given by (3.10) using the Höhler 8.2 parameterization[55] of the nucleon form factors. The dotted lines represent the Dirac PWIA calculation with the nuclear response tensor described by (3.13). The equivalent nonrelativistic DWIA calculations, as described in Section 3, is represented by the dashed lines. A calculation, which for convenience we refer to as "on shell", is represented by the dot-dashed lines. In this calculation, only the pole part of the propagator which appears in the Møller operator for the scattering wave function is kept. This forces the nucleon-nucleus scattering t matrix, which appears in this Møller operator, to be on shell. By comparison with the full Dirac and nonrelativistic calculations, this calculation can be used as a rough measure of the sensitivity of the DWIA calculations to the off-shell components of the t matrix which are not so highly constrained by experimental elastic proton scattering.

A careful examination of the eighteen response functions shown in Figs. 2-4 shows that there is no consistent relationship among the various calculations which holds for all of the response functions. This is not surprising since seven of the eighteen response functions cannot even contribute in the PWIA, but do so in the various distorted wave calculations. This diversity alone suggests that it is indeed likely that a selective separation of response functions may be useful in assessing the merits of various models applicable to this reaction. Although there seems to be no global relationship between the four calculations, some interesting patterns do appear in Figs. 2-4.

First we note from the figures that a number of the polarization response of the two transverse components of the transition current density. Much like $R_{LT'}$, the response function R_{LT}, which is predicted to be appreciable, also shows considerable sensitivity to off-shell effects while being totally insensitive to the difference between Dirac and nonrelativistic dynamics. Unlike $R_{LT'}$ however, R_{LT} is very large in the PWIA limit, so that its response to final state effects is very different from and complementary to that of $R_{LT'}$.

An important characteristic of these off-shell contributions to the scattering wave functions is that, since such contributions can not propagate to infinity, they are nonzero only in the nuclear interaction volume. Therefore, response functions which are very sensitive to the inclusion of the off-shell contributions are sensitive to the detailed characteristics of the scattering wave function in the nuclear interior. This has implications beyond the DWIA since it suggests that these response functions may also have the potential to be sensitive to many-body effects such as more sophisticated treatments of the nuclear wave function, exchange currents, and correlations, which modify the effective current operator in the nuclear interior.

For the large response functions the dynamical differences between the relativistic and nonrelativistic DWIA calculations result in differences in size of on the order of 5 to 10 percent, with the longitudinal response function R_L showing an enhanced effect of 15 to 20 percent. This apparent relativistic suppression of R_L relative to R_T is especially interesting in view of an analogous suppression which has been observed in inclusive electron scattering. For the smaller response functions the dynamical effects of relativity are on the order of 5 to 10 percent with the exceptions of R^n_{LT}, R^t_{LT} and R^l_{TT} where the effects vary from 20 to 35 percent, and R^n_L where the effect is 75 percent. Of course, small response functions are more likely to be very sensitive to variations in the details of computational models since they are small because of the cancellation of leading order contributions which dominate the large response functions. Anything which perturbs these cancellations can result in large changes in these small response functions. Thus, it should be noted that some of the small response functions such as R^n_T and large polarization response functions. This entails only in-plane measurements, the ability to flip the electron helicity, and a final ejectile spin determination.

The flexibility provided by final state polarization measurements in the $(\vec{e}, e'\vec{p})$ reaction is considerable. In our study, every variation we have considered produces distinctive implications for some subset of the response functions. As in the case of medium and off-shell current operator effects, there is every reason to expect this trend to continue as additional realistic physical processes are explored. For example, realistic nuclear structure implications, exchange currents, and further relativistic effects remain to be explored. Also, the four-momentum transfer behavior of the response functions needs to be explored, especially as a function of differing dynamical models. The results of the present study suggest that such investigations will prove interesting. It is also clear from this initial study that potential advantages to be gained by measuring these new response functions merit an investment in studies of the feasibility of separating some or all of these response functions from the cross section and in efforts to develop any new experimental techniques which may be necessary to achieve this goal.

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$$= \int_{line} dE' \, \left[W^{11}\left(\hat{s}'_R\right) - W^{22}\left(\hat{s}'_R\right)\right]$$

$$\frac{1}{2}\left[\left(R_{LT} + R^n_{LT} S_n\right)\cos\beta + \left(R^l_{LT} S_l + R^t_{LT} S_t\right)\sin\beta\right]$$

$$= - \int_{line} dE' \, \left[W^{01}\left(\hat{s}'_R\right) + W^{10}\left(\hat{s}'_R\right)\right]$$

$$\frac{1}{2}\left[\left(R_{LT'} + R^n_{LT'} S_n\right)\sin\beta + \left(R^l_{LT'} S_l + R^t_{LT'} S_t\right)\cos\beta\right]$$

$$= i \int_{line} dE' \, \left[W^{20}\left(\hat{s}'_R\right) - W^{02}\left(\hat{s}'_R\right)\right]$$

$$\frac{1}{2}\left(R^l_{TT'} S_l + R^t_{TT'} S_t\right) = i \int_{line} dE' \, \left[W^{12}\left(\hat{s}'_R\right) - W^{21}\left(\hat{s}'_R\right)\right].$$

These definitions yield β-independent response functions consistent with the expression for the cross section (2.1). The relationship between these "new" response functions and the "old" ones can be written as

$$R^i_j = C^i_j \left(R^i_j\right)_{old} \tag{A5}$$

where no summation of indices is implied. The coefficients C^i_j are given by

$$\begin{aligned}
C_L &= C^n_L = 1 \\
C_T &= C^n_T = 1 \\
C_{TT} &= C^n_{TT} = C^l_{TT} = C^t_{TT} = -1 \\
C_{LT} &= C^n_{LT} = -1, \; C^l_{LT} = C^t_{LT} = 1 \\
C_{LT'} &= C^n_{LT'} = 1, \; C^l_{LT'} = C^t_{LT'} = -1 \\
C^l_{TT'} &= C^t_{TT'} = 1.
\end{aligned} \tag{A6}$$

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51. The most general Lorentz four-vector is constructed by combining the complete set of independent Dirac γ-space tensors $(1,\ \gamma_5,\ \gamma^\mu,\ \gamma^\mu\gamma_5,\ \sigma^{\mu\nu})$ with a corresponding set of momentum-space tensors constructed from two independent momenta p^μ and p'^μ, and the Levi-Civita tensor density $\epsilon^{\mu\nu\rho\sigma}$. Since the γ-space tensors are of rank two or lower, the momentum-space tensors needed are at most of rank three. Twenty four four-vectors are obtained, half of which do not transform properly under parity. The remaining set of twelve independent four-vectors, each of which is multiplied by an arbitrary scalar function of the four-momenta, and forming the general form of the current operator, are: p^μ, p'^μ, γ^μ, $\not{p}'p'^\mu$, $\not{p}p^\mu$, $\not{p}'p^\mu$, $\not{p}p'^\mu$, $\gamma_5\epsilon^{\mu\nu\rho\sigma}\gamma_\nu p'_\rho p_\sigma$, $\sigma^{\mu\nu}p'_\nu$, $\sigma^{\mu\nu}p_\nu$, $p'^\mu\sigma^{\alpha\beta}\left(p_\alpha p'_\beta - p'_\alpha\, p_\beta\right)$ and

Table Captions

1. Properties of Response Functions

Figure Captions

1. Coordinate system used in discribing the $(\vec{e}, e'\vec{N})$ reaction.
2. Response functions for the ejection of a $T_{p'} = 135\,MeV$ proton from the $1p_{1/2}$ shell of ^{16}O. The response functions are shown for fixed momentum transfer $|\vec{q}| = 2.641\,fm^{-1}$ as a function of the magnitude of the recoil momentum. The solid and dashed lines represent the relativistic and nonrelativistic DWIA calculations, while the dotted line represents the relativistic PWIA and the dot-dashed line represents the relativistic "on-shell" calculation, as described in the text.
3. Same as Fig. 2.
4. Same as Fig. 2.
5. The hatched region represents the physically available values of momentum transfer and recoil momentum for the $(\vec{e}, e'\vec{N})$ reaction. Parallel/Antiparallel kinematics cooorespond to the borders of this region, while the kinematics used in Figs. 2-4 is represented by the dashed line.

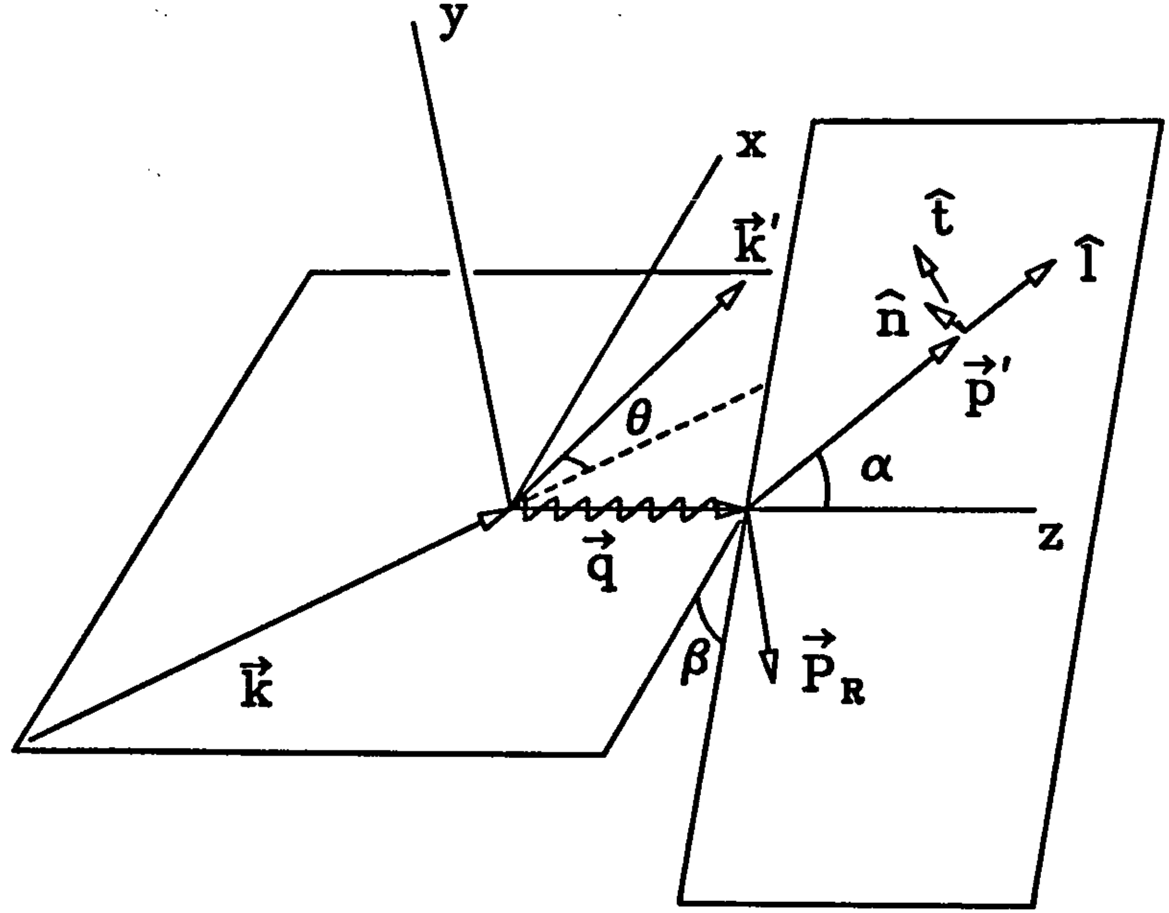
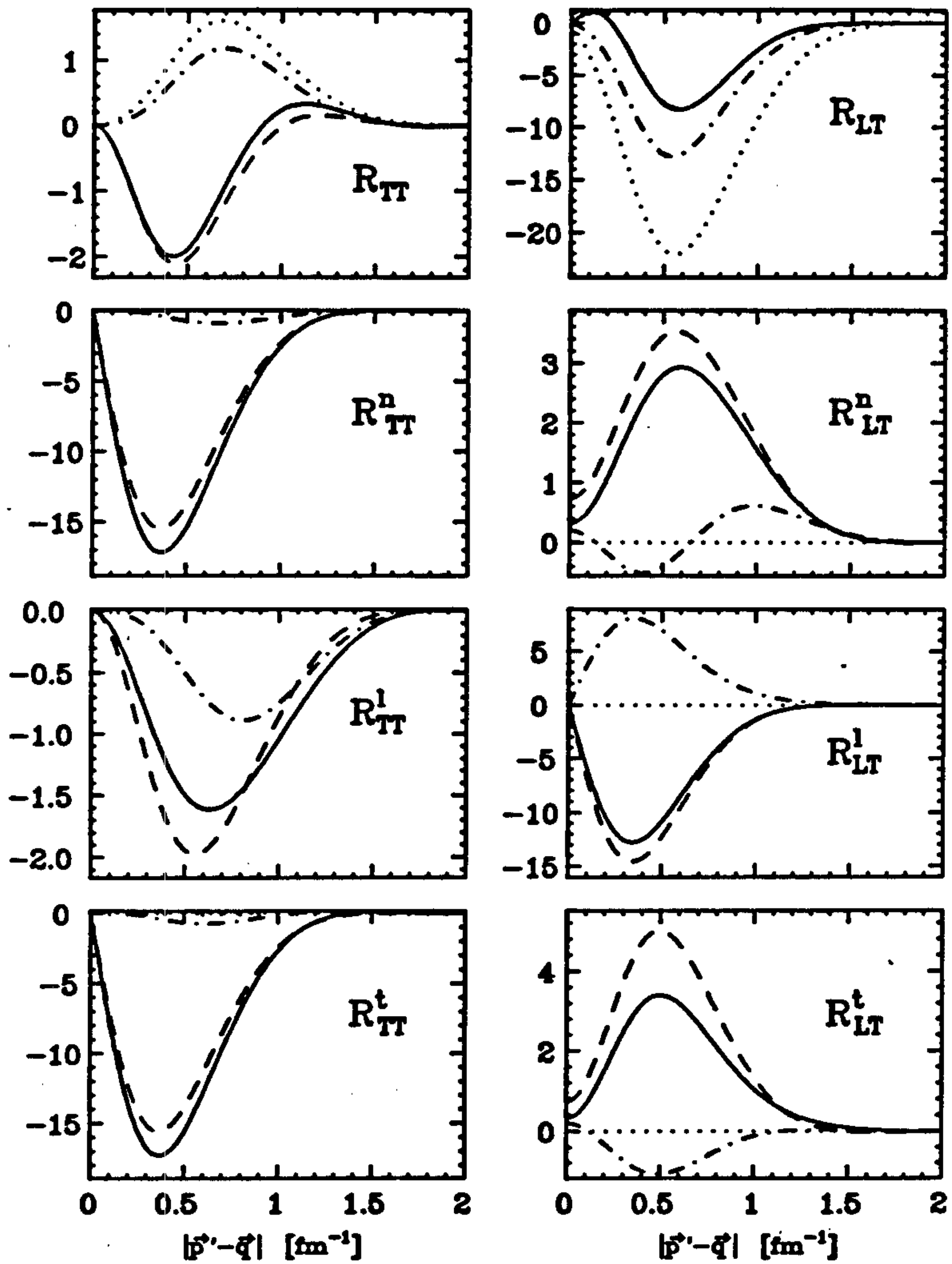

y
x
$\vec{k}'$
$\hat{t}$
$\hat{l}$
$\hat{n}$
$\vec{p}'$
θ
α
z
$\vec{q}$
β
$\vec{P}_R$
$\vec{k}$

Figure 1



R_{TT}
R_{LT}
R^n_{TT}
R^n_{LT}
R^l_{TT}
R^l_{LT}
R^t_{TT}
R^t_{LT}
$|\vec{p}\,'-\vec{q}|$ [fm^{-1}]
$|\vec{p}\,'-\vec{q}|$ [fm^{-1}]

Figure 3

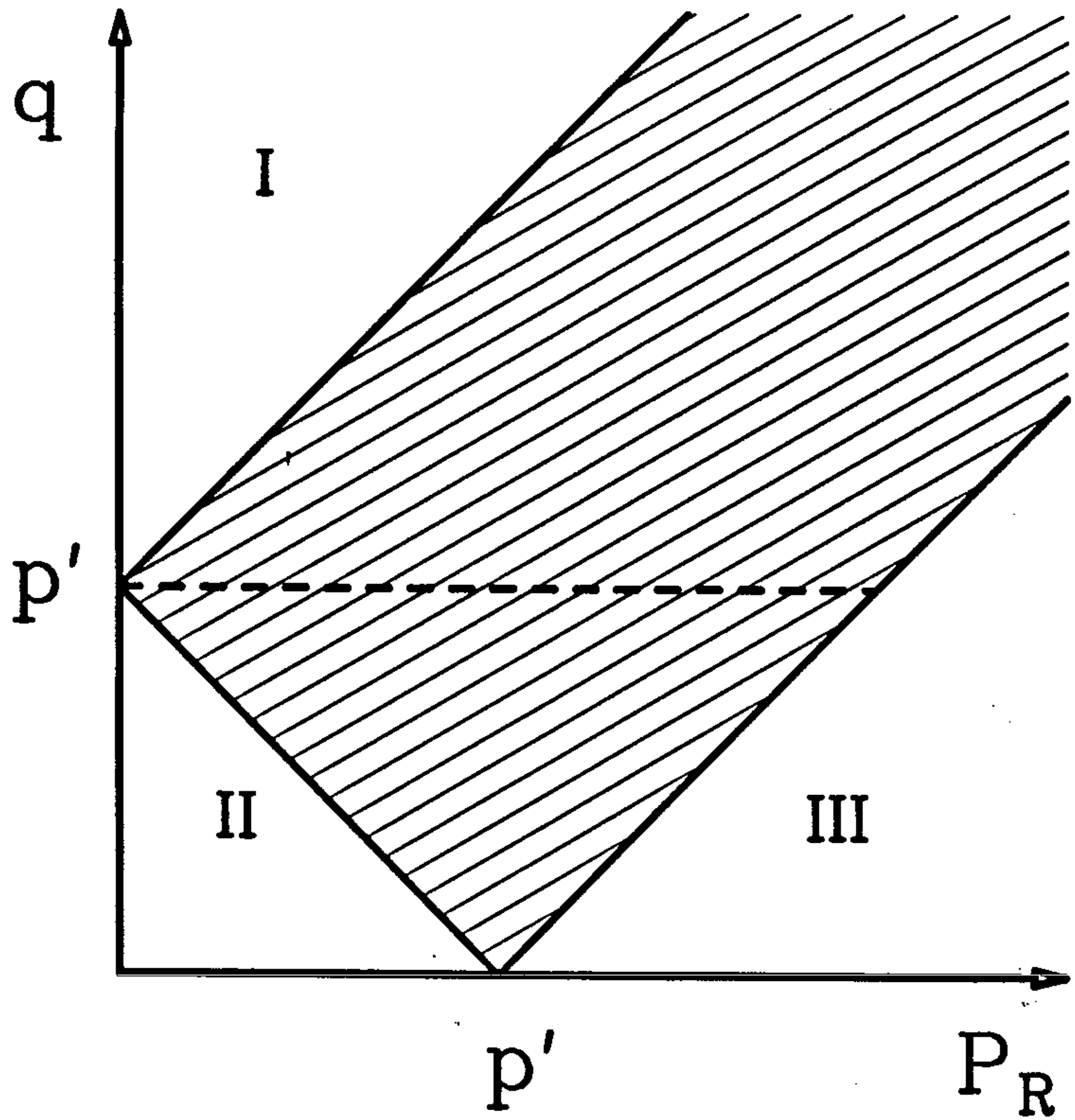


Figure 5